

On a parameterized family of relative Thue equations

CATRIN LAMPL¹ <catrin.lampl@unileoben.ac.at>

Let $k := \mathbb{Q}(\sqrt{-D})$ be an imaginary quadratic number field and \mathbb{Z}_k be the corresponding ring of integers. We consider the family of relative Thue equations

$$F_t(x, y) = x^3 - (t-1)x^2y - (t+2)xy^2 - y^3 = \ell \quad (1)$$

with $t, \ell \in \mathbb{Z}_k, t \notin \mathbb{Z}$ and $|\ell| \leq |2t+1|$. Let $k(\alpha)$ be the cubic extension of k generated by a root α of the polynomial $f_t(x) = F_t(x, 1)$ and let $\mathbb{Z}_{k(\alpha)}$ be its ring of integers. Let (x, y) with $x, y \in \mathbb{Z}_k$ be a solution of (1). We determine all elements $\gamma = x - \alpha y \in \mathbb{Z}_{k(\alpha)}$ whose norms satisfy $|N_{k(\alpha)/k}(\gamma)| \leq |2t+1|$ and solve the Thue equations for all $t \in \mathbb{Z}_k, t \notin \mathbb{Z}$ with $\Re t = -\frac{1}{2}$ and all $|\ell| \leq |2t+1|$. Supported by the Austrian Science Foundation (FWF) Project Nr. S8310.

¹Montanuniversität Leoben