Rigorous bounds on the Nusselt number

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We are interested in the transport of heat through a layer of viscous fluid which is heated from below and cooled from above. Two mechanisms are at work: Heat is transported by simple diffusion and by advection through the flow. The transport by advection is triggered by buoyancy (hotter parts have lower density) but is hindered by the no–slip boundary condition for the fluid velocity at the bottom and top surfaces.

Neglecting inertia, the equations contain a single dimensionless parameter, the Rayleigh number Ra. It measures the relative strength of advection with respect to diffusion. For $Ra \gg 1$, the flow is aperiodic and the heat transport is mediated by plumes. As a consequence, the horizontally averaged temperature displays boundary layers.

Inspired by the work of Constantin and Doering, we are interested in rigorous b ounds on the average heat transport (the Nusselt number Nu) in terms of Ra. By PDE methods, Constantin and Doering prove $Nu \lessapprox Ra^{1/3} \log^{2/3} Ra$.

We use the conceptually intriguing method of the background (temperature) field, introduced by Hopf for the Navier-Stokes equation and used by Temam et. al. for the Kuramoto–Sivashinski equation. We propose a background temperature field with monotone boundary layers; direct numerical simulations show an average temperature field with the same qualitative behavior. We obtain the slightly improved bound $Nu \lessapprox Ra^{1/3} \log^{1/3} Ra$.
. The crucial ingredient is a maximal regularity statement for the Stokes operator in suitably weighted L^2–spaces.

This is joint work with Charlie Doering and Maria Reznikoff.

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