Classes of Operators with Fixed Points on Hilbert Spaces

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Let H be a real Hilbert space with inner product $<,>$, and consider the class $\mathcal{I}$ of operators $T : H \to H$ such that each T has a nonempty set Fix$T$ of fixed points, and such that $< x - Tx, z - Tx > \leq 0$, for each x in H and for each z in Fix$T$ (see [1]). We first present a parallel algorithm to find, in a finite number of steps, a common fixed point of a finite number of operators from this class when it is known that the intersection of their fixed point sets contains an interior point. Next, we remark that the class $\mathcal{I}$ can be imbedded in a hierarchical system of classes of operators depending on a nonnegative real parameter ν; the classes in this system are denoted by QNE(ν, H) (shorthand for ν-quasi-nonexpansive on H). When μ and ν are nonnegative real numbers and $\mu > \nu$, then QNE(μ, H) is included in QNE (ν, H). We comment on the possibility of extending results that formerly have been proved for operators belonging to the class QNE$(1, H)$, to the class QNE (ν, H) with $0 \leq \nu < 1$.

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