On Geometry of Regular Heptagons

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In geometry regular polygons (the equilateral triangles, the squares, the regular pentagons,...) have special appeal and significance. With the help of computers today it is possible to find their new fascinating properties.

We start from a classical result by Thébault and Demir that six midpoints of sides and diagonals of a regular heptagon $ABCDEFG$ lie on a cirlce m with diameter equal to the side of the square inscribed in the circumcircle of $ABCDEFG$. Then we describe eight more midpoints of segments on m. Extending further this idea we show that midpoints of many other segments also lie on the circle m so that it deserves the name – the *midpoints circle* of $ABCDEFG$.

Then we present two more circles n and p associated to a regular heptagon $ABCDEFG$. Their centers are the incenter and the excenter of the triangle ABG and they go through the incenter and the excenter of the triangle DEB, respectively. These circles contain intersections of many lines which are often vertices of new regular heptagons related to $ABCDEFG$.

Finally, we look at the central points (like incenters, centroids, circumcenters and orthocenters) of certain triangles in the regular heptagon to find new related regular heptagons which have simple constructions with ruler and compass from the original heptagon. In the proofs we use the complex numbers and the software Maple V. The figures and some of our discoveries are made with the Geometer's Sketchpad.