Entropy Methods for Systems Combining Diffusion and Nonlinear Reaction

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Reaction-diffusion systems for chemicals, drift-diffusion and recombination in semiconductors, coagulation and fragmentation of polymers are examples from models which combine diffusion and nonlinear reactions in terms of an entropy (free energy) functional. We present entropy methods as physically intuitive and well appropriate to analyse the existence of global solutions and their long-time behaviour. It is a major advantage of the entropy method to be quite robust. This is due to the fact that it mainly relies on functional inequalities which have no direct link with the original PDE.

We present in particular reaction-diffusion systems on bounded domains modelling chemical substances with individual diffusivities, which react in a reversible way according the principle of mass-action kinetics. The considered examples feature unique steady states, which can be characterised as minimising states of the non-increasing entropy functional. The production rate of the entropy functional - the so called entropy dissipation - combines the effects of the diffusion- as well as of the reaction process.

We compute explicit bounds on the rates for the exponential convergence of solutions towards the steady state in two situations of degeneracy: Firstly, for a two species system, when spatial diffusion of one specie vanishes. Secondly, for a system of four species in 1D, we deduce 1) an at most polynomially growing L^∞-bound from a-priori-estimates on the entropy and entropy dissipation, 2) almost exponential convergence to the steady state via a precise entropy-entropy dissipation estimate, 3) an explicit global L^∞-bound via interpolation of a polynomially growing H^1-bound with the almost exponential L^1-convergence, and 4), finally, explicit exponential convergence to the steady state in all Sobolev norms.

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