HAMILTONICITY OF CUBIC CAYLEY GRAPHS

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In 1969, Lovász asked whether every connected vertex-transitive graph has a Hamilton a path, thus tying together, through this special case of the Traveling Salesman Problem, two seemingly unrelated concepts: traversability and symmetry of graphs. Lovász problem is, a somewhat misleadingly, usually referred to as the Lovász conjecture, presumably in view of a the fact that, after all these years, a connected vertex-transitive graph without a Hamilton path is yet to be produced.

In this talk I will give a quick overview of a recent result showing that Lovász conjecture a is true for cubic Cayley graphs arising from groups having a $(2, s, 3)$-presentation, that is, for groups $\langle G = a, b | a^2 = 1, b^s = 1, (ab)^3 = 1, etc.\rangle$ generated by an involution a and an element b of order $s \geq 3$ such that their product ab has order 3. More precisely, every Cayley graph $X = Cay(G, a, b, b^{-1})$ has a Hamilton cycle when $|G|$ (and thus s) is congruent to 2 modulo 4, and has a long cycle missing only two vertices (and thus necessarily a Hamilton path) when $|G|$ is congruent to 0 modulo 4.

This is a joint work with Henry Glover.

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