Maximum independent vertex sets in hamiltonian 4-regular graphs

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As had been conjectured by P. Erdös and was proved by M. Stiebitz and the speaker, cycle-plus-triangles graphs are 3-colourable (they are even 3-choosable). However, if one considers a hamiltonian 4-regular graph G decomposable into a hamiltonian cycle H and conformly inscribed cycles (that is, a run through a component of $G - H$ corresponds to a subsequence of $V(H)$ if one traverses H in a fixed direction), then 3-colourability is an NP-complete problem, and the same conclusion holds if one just asks for an independent set of order $n/3$ where n is the order of G. On the other hand, one can easily prove that the independence number $\alpha(G)$ is at least $(n - r)/3$ where r is the number of components in $G - H$.

Considering from among the above graphs only those that have no independent set of size at least $n/3$, one can write for these graphs

$$\alpha(G) = (n - cr)/3 \tag{1}$$

where n and r are as above, and c lies in the interval $(0, 1]$. It turns out that for every rational c in this interval there exist $n = n(c)$ and $r = r(c)$ and a 4-regular graph G of order n decomposable into a hamiltonian cycle H and r conformly inscribed cycles such that G satisfies Equation 1.

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