# Rough Paths - a top down description of controls

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The theory of rough paths as developed by the Author (and several others such as Hambly, Ledoux, Coutin, Qian, Fritz, ...) aims to study the differential equations used to model the situation where a system responds to external control or forcing. The theory describes a robust approach to these equations that allows the forcing to be far from differentiable. The methodology permits the main probabilistic classes as well as many new types of stochastic forcing that do not fit into the classical semi-martingale setting directly.

The key to this theory is to answer the question - when do two controls produce similar responses. This is also a core question for the problem for multi-scale analysis where one needs to summarise small scale behaviour in a way that large scale responses can be predicted from the summarised information. The question can be translated into one asking that one characterises the continuity properties of the Itô map. This is indeed possible and the Universal Limit theorem proves the (uniform) continuity of the map taking the forcing control to response for a wide class of metrics on smooth paths - and the completions of the space under these metrics give the so called rough paths - giving insight into the control problem.

The approach is quite structured, and allows one to give a top down analysis of a control in terms of a sequence of algebraic coefficients we call the signature of the control (which have similarity to a child's précis of a complicated text by a simpler one and are a non-commutative analogue of Fourier coefficients) with refinements giving more accurate information about the control. Hambly and Lyons recently proved that this "signature" of a control completely characterises the control up to the appropriate null sets.

The new results mentioned above have generated new open problems

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