The distribution of patterns in random trees

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Let $\mathcal{T}_n$ denote the set of unrooted unlabeled trees of size n and let $\mathcal{M}$ be a particular (finite) tree. Assuming that every tree of $\mathcal{T}_n$ is equally likely, it is shown that the number of occurrences X_n of $\mathcal{M}$ as an induced sub-tree satisfies $\mathbf{E}X_n \sim \mu n$ and $\mathbf{V}ar\, X_n \sim \sigma^2 n$ for some (computable) constants $\mu > 0$ and $\sigma \geq 0$. Furthermore, if $\sigma > 0$ then $(X_n - \mathbf{E}X_n)/\sqrt{\mathbf{V}ar\, X_n}$ converges to a limiting distribution with density $(A + Bt^2)e^{-Ct^2}$ for some constants A, B, C. However, in all cases in which we were able to calculate these constants, we obtained $B = 0$ and thus a normal distribution. Further, if we consider planted or rooted trees instead of $\mathcal{T}_n$ then the limiting distribution is always normal. Similar results can be proved for planar, labeled and simply generated trees.

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