On some parametric families of quartic Thue equations and related familiy of relative Thue equations

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Solving of the two-parametric family of quartic Thue equations

$$x^4 - 2mnx^3y + 2\left(m^2 - n^2 + 1\right)x^2y^2 + 2mnxy^3 + y^4 = 1, \tag{1}$$

using the method of Tzanakis, reduces to solving the system of Pellian equations $V^2 - \left(m^2+2\right)U^2 = -2$, $Z^2 - \left(n^2 - 2\right)U^2 = 2$. We show that if $|m|$ and $|n|$ are sufficiently large and have sufficiently large common divisor, then the system has only the trivial solutions, which implies that the original Thue equation also has only the trivial solutions. Further, we prove that for all integers m and n there are no non-trivial solutions of equation (1) satisfying the additional condition $\gcd(xy, mn) = 1$. We will also show that system of Pellian equations for $n \neq 0, \pm 1$ possess at most 7 solutions in positive integers. The case $m = 2n$ can be considered as a special case of the Thue equation

$$x^4 - 4cx^3y + (6c+2)x^2y^2 + 4cxy^3 + y^4 = 1,$$

which is completely solved. We also consider the related relative Thue equation

$$x^4 - 4cx^3y + (6c+2)x^2y^2 + 4cxy^3 + y^4 = \mu,$$

where the parameter c and the root of unity μ are integers in the same imaginary quadratic number field. We show that for $|c| > 4$ only certain values of μ yield solutions of this equation and solve equation completely under the assumption $|c| \geq 1544686$.

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