

SIERPIŃSKI CURVES AND UNIVERSAL SPACES

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Two planar curves bear W. Sierpiński's name, a "triangular" one (also called Sierpiński gasket) and a "square" one (called Sierpiński carpet). The "triangular" curve was published in 1915 and was the first example of a space whose almost every point is a ramification point. The "square" curve appeared in 1916, when Sierpiński proved that it contains a homeomorphic image of every planar curve, making it the first example of a universal space. The "square" curve was later generalized by K. Menger to a whole sequence of spaces (today called Menger compacta). He proved that a curve constructed analogously in the 3-dimensional cube is universal for all 1-dimensional separable metrizable spaces. In this way, Sierpiński can be considered to be the originator of universal spaces. Generally, if a class of spaces is given, a space is called universal for the given class, if it belongs to the class and every space of this class can be topologically embedded into it. The class considered in this talk is the class of metrizable spaces of dimension less than or equal n .

In the talk, we shall present how Sierpiński's "triangular" curve comes into play again by generalizing his curve to one of any given weight, and how the original "triangular" Sierpiński curve can be used in the separable case. Furthermore, we wish to present two recent results of relative type, i.e. to consider the situation where an embedding in a universal space is already given on a subspace and we want to extend it to an embedding of the whole space.

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