Parameter Estimation for Fokker-Planck Equation with Application to Non-linear Exchange Rate Dynamics

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It is common practice in financial modeling that the price dynamics S is modeled by an Itô stochastic differential equation:

$$dS = \mu(t, S(t), Z(t))dt + \sigma(t, S(t), Z(t))dW. \tag{1}$$

Here $Z(t)$ are external, such as economic or political effects and W is a standard Wiener process with the property that dW is distributed as $\mathcal{N}(0, dt)$, and μ and σ satisfy Lipschitz and growth conditions sufficient for the existence of a continuous solution to (1). In case that all necessary coefficients of the model equation are known, solutions to (1) can be computed using available algorithms. However, in reality the drift term $\mu(\cdot)$ and the volatility $\sigma(\cdot)$ are unknown and need to be determined by modeling and/or by solving a a special parameter estimation problem. First, we assume that the drift term and the volatility function of the stochastic differential equation (1) depend on the spatial variable s and unknown parameter vector θ: $\mu = \mu(t, s; \theta)$, $\sigma^2 = \sigma^2(t, s; \theta)$. Using the fact that the transitional price distribution $f(t, s)$ of the stochastic process S_t at each point of time satisfies the forward Kolmogorov equation, we estimate the unknown parameters by solving the parameter estimation problem for forward Kolmogorov equation. Taking into account nonlinear effects in volatility and drift and dependence on economical data, which are not directly modelled, one obtains equations where the standard numerical methods are not sufficient. The coefficients are rapidly oscillatory, strong instabilities may arise.

We present new efficient algorithms to identify parameters for Kolmogorov equation and to simulate the dynamics of exchange rate depending on economic data.

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