Central Limit Theorems for Poisson Hyperplane Tessellations

LOTHAR HEINRICH[1] <heinrich@math.uni-augsburg.de>
VOLKER SCHMIDT[2] <schmidt@mathematik.uni-ulm.de>
HENDRIK SCHMIDT[3] <hendrik@mathematik.uni-ulm.de>

We consider homogeneous and in particular motion-invariant Poisson hyperplane tessellations of the Euclidean space R^d which is observable in a ball B_r centered at the origin with radius r assumed to grow to infinity. We first derive a central limit theorem (briefly:CLT) for the total number of vertices of convex polytopes in B_r induced by a stationary d-dimensional hyperplane process. This CLT generalizes an earlier result proved by K.Paroux [1] for intersection points of a motion-invariant planar Poisson line process. Our proof is based on Hoeffding's decomposition technique for U-statistics with a Poisson distributed sample size which seems to be more efficient and adequate to tackle the higher-dimensional case than the 'method of moments' used in [1] to treat the case $d = 2$.
Moreover, we extend our CLT in several directions. First, we investigate all the k-flat processes (for $1 \leq k \leq d-1$) induced by $(d-k)$-tuples of intersecting Poisson hyperplanes and derive asymptotic normality of the number of such k-flats hitting B_r as well as of their total k-volume contained in B_r. If in addition the Poisson hyperplane process is isotropic the asymptotic variances occurring in the Gaussian limit laws can be calculated explicitly by means of Crofton's formula. These results are used to construct asymptotic confidence intervals for the intensities of the k-flat intersection processes.
In a second step we prove multivariate CLTs for the suitably normalized joint vectors of the number k-flats resp. of their k-volumes (for $k = 0, 1, ..., d-1$) in B_r as $r \to infty$. It turns out that in the motion-invariant case the asymptotic covariance matrix of the numbers of k-flats hitting B_r possesses full rank d, whereas the corresponding covariance matrix of the total k-volumes in B_r has rank 1 for any $d \geq 2$.

[1] K. Paroux: *Quelques thèorém centraux limites pour les processus poissoniens de droites dans le plan* , Adv.Appl. Probab. 30, 640-656

[1]Universitaet Augsburg, Institut fuer Mathematik
[2]Universitaet Ulm
[3]Universitaet Ulm