Diophantine m-tuples and connections with elliptic curves

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A Diophantine m-tuple is a set of m positive integers such that the product of any two of them is one less than a perfect square. If the elements of such set are non-zero rationals, then we call it a rational Diophantine m-tuple. It is known there does not exist a Diophantine sextuple and that there are only finitely many Diophantine quintuples, but there exist rational Diophantine sextuples. Many properties of Diophantine triples $\{a, b, c\}$ and quadruples $\{a, b, c, d\}$ can be naturally interpreted in terms of the corresponding elliptic curves $y^2 = (x + ab)(x + ac)(x + bc)$, $y^2 = x(x + (b - a)(d - c))(x + (c - a)(d - b))$. On the other hand, the known results on extensibility of Diophantine triples can give us information about integer points on some families of elliptic curves, as it was done in our joint work with A. Pethő.

In this talk, we will also mention some applications of Diophantine m-tuples in the construction of elliptic curves with relatively high rank and large torsion group.

Finally, we will describe our recent result concerning the existence of infinitely many rational $D(q)$-quintuples (quintuples of rationals such that $xy + q$ is a perfect square for all its distinct elements x, y) for some rationals q. Namely, for those q's for which the corresponding twist of a particular elliptic curve has positive rank. The construction of such quintuples is related to our recent joint papers with C. Fuchs, R. Tichy and G. Walsh on Diophantine m-tuples for linear polynomials.

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