Multiscale Solutions for the Poisson Equation on the 3-dim. Ball

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Within the talk at hand, we investigate the Poisson equation

$$\Delta_x V(x) = \rho(x)$$

for the gravitational potential V, corresponding to the density distribution ρ of the ball-shaped Earth B_R. Its solution can at least for Hölder-continuous densities, be solved by the operator

$$\begin{aligned} T : L_2\left(\overline{B_R}\right) &\rightarrow L_2\left(\overline{B_R}\right) \\ \rho &\mapsto T\rho := \int_{\overline{B_R}} \frac{\rho(y)}{|\cdot - y|} dy \end{aligned}$$

originating from an ansatz by Greens functions. This connection between mass distributions and the gravitational force is essential to investigate, especially inside the Earth, where structures and phenomena are not sufficiently known and plumbable.

Since the operator stated above does not solve the equation for all square-integrable functions, the solution space will be decomposed by a multiscale analysis in terms of scaling functions. Due to geometry of the region under consideration, classical Euclidean wavelet theory appears not to be the appropriate choice. For this reason, these ansatz functions are chosen to be reflecting the rotational invariance of the ball. In these terms, the operator itself is finally decomposed and replaced by versions more manageable. This decomposition is able to reflect the structure of the operator of resolution in detail.

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