Wavelets on the 3–dimensional Ball

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Standard tools for modelling an unknown function on the 3–dimensional ball are the expansion in orthogonal polynomials and the subdivision of the ball in blocks where functions of simple structure (e.g. constant or polynomial) are used per block. The disadvantage of the first approach is its global character. Spatially varying data density or quality cannot be taken into account appropriately. Moreover, local modifications, e.g. due to temporal effects such as earthquakes or seasonal processes in case of the Earth, cannot be realized. On the other hand, the block ansatz only yields a discrete model with limited resolution.
In this talk it is shown how orthogonal polynomials can be used to construct localizing kernels on 3–dimensional balls that allow a multiresolution analysis including all typical features of wavelets. The advantage is that we are enabled to model functions on a ball with locally varying resolution but by a continuous and not discrete model. Moreover, this approach allows the multiscale solution of various equations such as the Laplace equation, the Poisson equation and certain Fredholm integral equations of first kind. This is illustrated by some numerical results.
Finally, aspects of finding optimally localizing wavelets on a sphere or a ball in $\mathbb{R}^3$ are discussed. Such wavelets promise to have numerous applications e.g. in geosciences and in medicine.

[1] V. Michel: *Wavelets on the 3-dimensional Ball*, Schriften zur Funktionalanalysis und Geomathematik (Preprint)
[2] V. Michel: *Regularized Multiresolution Recovery of the Mass Density Distribution from Satellite Data of the Earth's Gravitational Field*, Inverse Problems, 21 (2005), 997-1025

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